The Global Positioning System

Principles of GPS positioning
GPS signal and observables
Errors and corrections
Processing GPS data
GPS measurement strategies
Precision and accuracy

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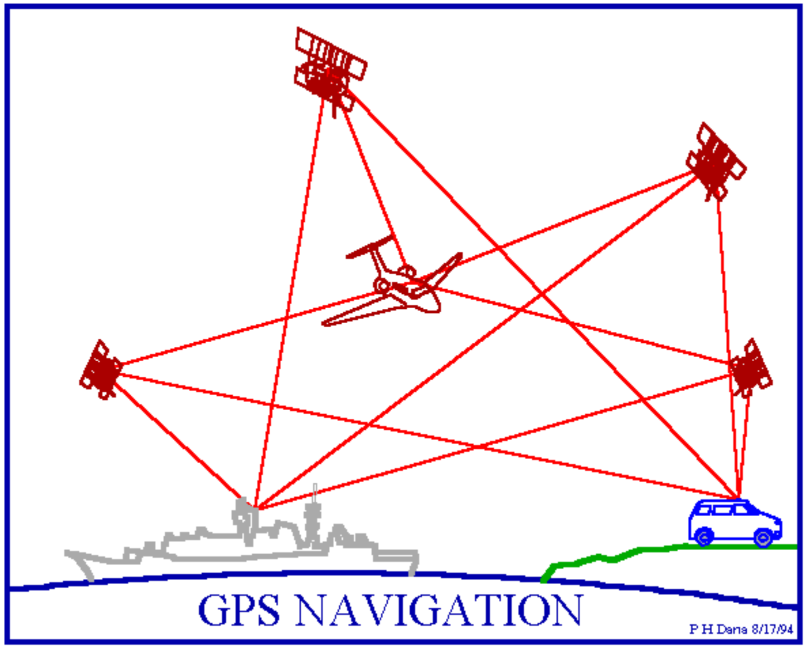
The Global Positioning System (GPS)

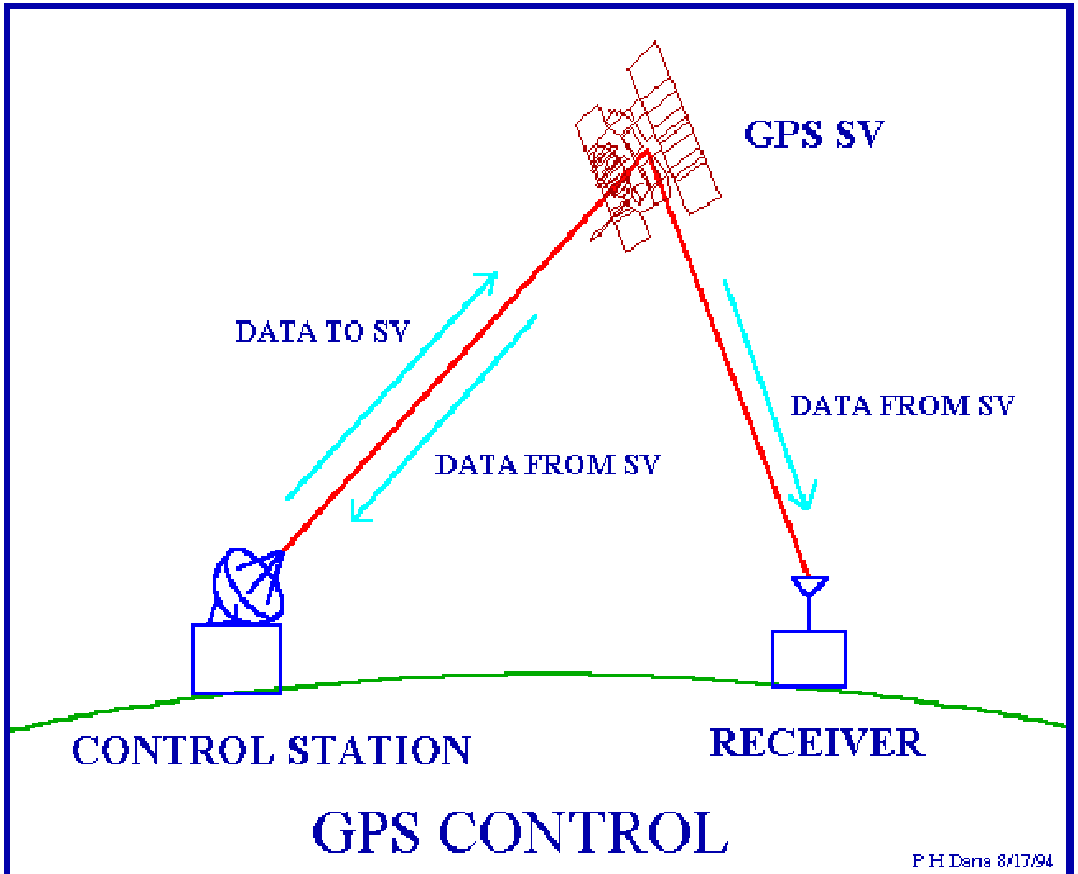


- A satellite-based positioning system available 24/24h everywhere on the globe with an accuracy better than 100 m.
- Originally designed for navigation and real-time positioning (meter-level accuracy): navigation (airplanes, ships, car, missiles, etc…)

It is **also capable of mm-level accuracy**, with important scientific "by-products":

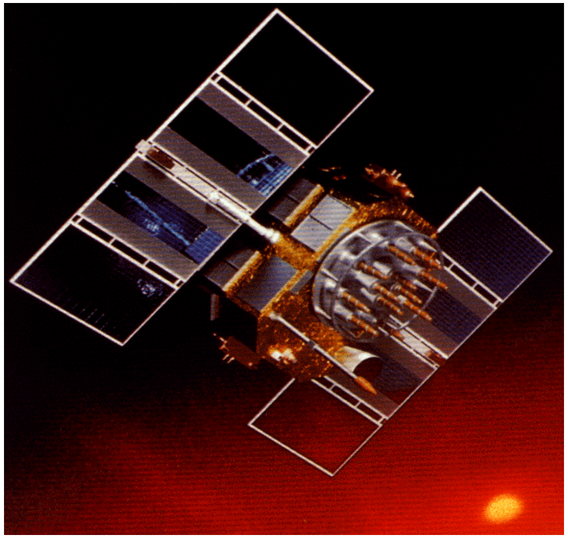
- ✓ In geodesy: shape and rotation of the Earth, terrestrial reference frame
- ✓ In solid Earth geophysics: deformation of the Earth's crust (earthquakes, volcanoes, plate tectonics)
- ✓ In atmospheric sciences: tropospheric water vapor, ionospheric electron content

Three "segments"

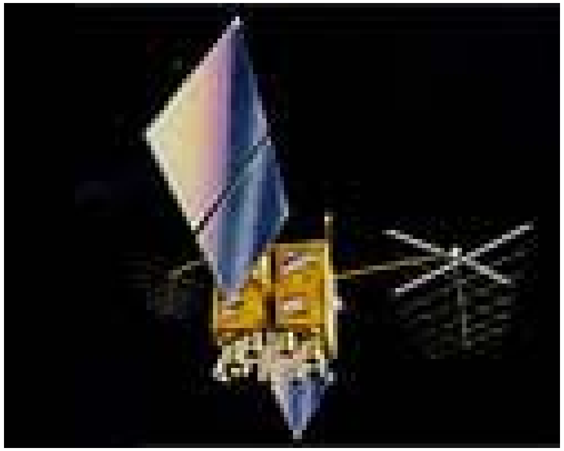


- The **space segment** = satellites:
  - Broadcast radio signals toward users on the Earth
  - Receive commands from the ground.
- The **control segment**: monitors the space segment and send commands to satellites
- The **user segment**: receivers record and interpret the radio signals broadcast by the satellites

The GPS satellites



Block II satellite

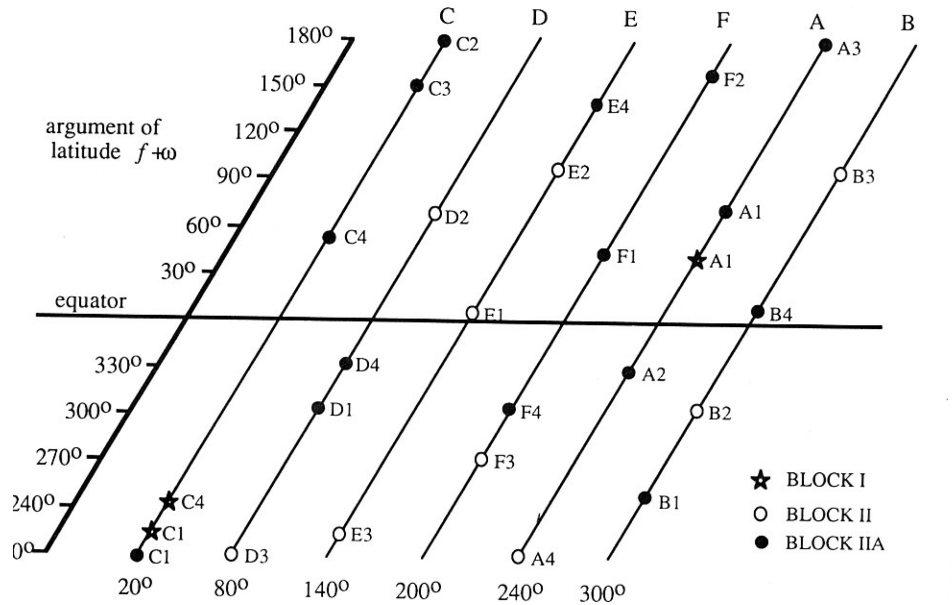


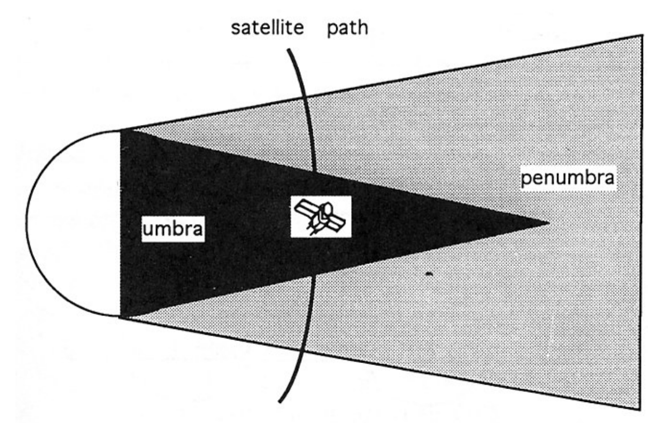
Block IIR satellite

- Four classes (=generations): blocks I, II, IIA, IIR,and IIF:
  - Block I:
    - 11 satellites launched between 1978 and 1985 on Atlas F rockets
    - Life expectancy = 4.5 years, actual mean life = 7.1 years
    - Signal entirely accessible to civilian users
    - Last block I satellite died on Feb. 28, 1994
  - Block II (II-R and II-F):
    - Possibility to degrade the signal for civilian users
    - 1 satellite ~ 25 million dollars
    - Life expectancy = 10 years
    - 5 m^3, 2 tons, solar panels, boosters
- New launches on a regular basis
- Monitored and controlled from the ground

Orbital constellation

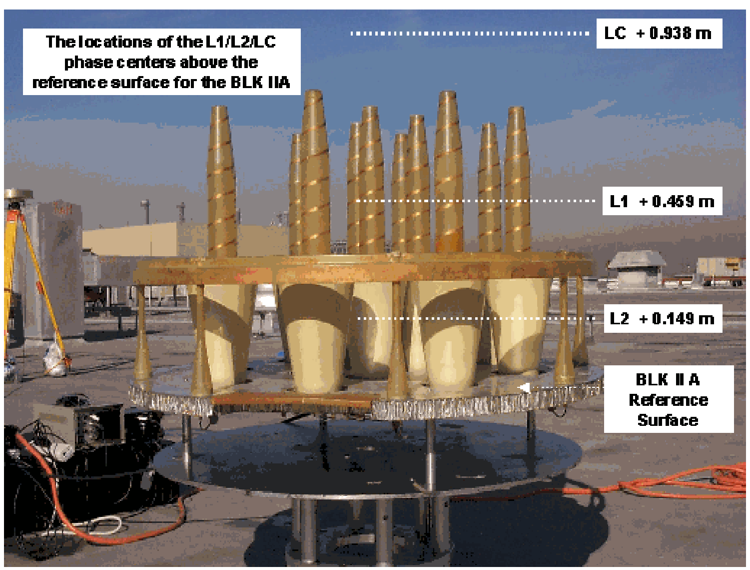
- 27 satellites (24 operational + 3 spares)
  - Quasi-circular orbits, mean altitude 20200 km
  - 6 evenly spaced orbital planes (A to F), inclination 55°
  - 4-6 satellites per plane, spacing for optimized visibility
  - Period = 12 sidereal hours (= 11h58mn "terrestrial" hours) ⇒ in a terrestrial frame, the constellation repeats every 23h56mn.
  - As Earth orbits around the Sun ⇒ eclipse periods (solar radiation pressure = 0, transition to shadow difficult to model, often simply edited out)
- In practice, 6-12 satellites are visible simultaneously, depending on:
  - Constellation geometry
  - Elevation cut-off angle (chosen by the user)



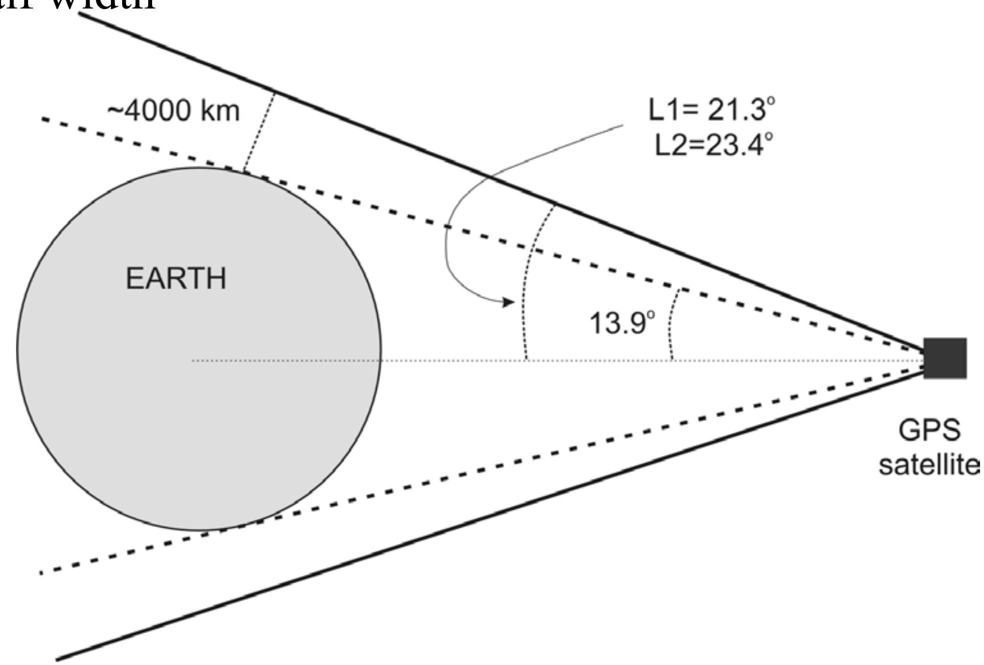


Satellite transmissions

- GPS satellites broadcast continuously on 2 frequencies in the L-band
- Future: GPS III, 3rd frequency
- GPS antennas point their transmission antenna to the center of the Earth
- Main beam = 21.4/23.4 (L1/L2) half width



Transmission antenna of a block II-R GPS satellite



GPS transmission beams

Satellite clocks

- Frequencies broadcast by GPS satellites are derived from a **fundamental frequency of 10.23 Mhz**

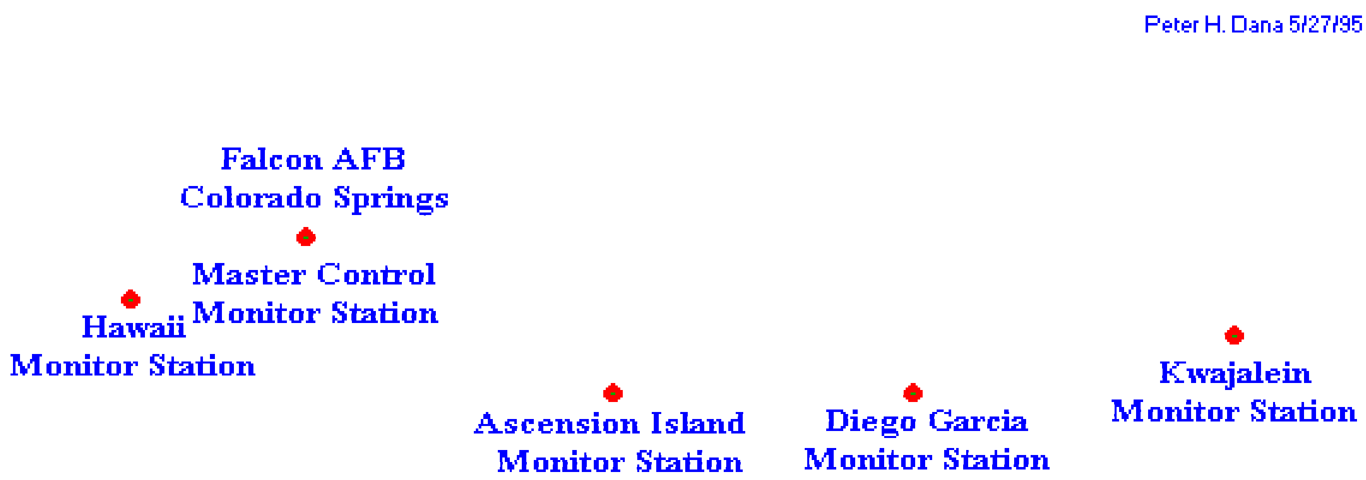
- Fundamental frequency provided by 2 or 4 atomic clocks (Ce/Rb)
  - Clocks run on GPS time = UTC not adjusted for leap seconds
  - Clock stability over 1 day = 10^{-13} (Rb) à 10^{-14} (Ce), ~ 1 ns/jour
  - Clocks **synchronized between all satellites**

- Relativistic effects:
  - Clocks in orbit appear to run faster (38.3 μsec/day = 11.5 km/day!) ⇒ tuned at 10.22999999543 MHz before launching (g.)
  - Clocks speed is a function of orbit eccentricity (45 nsec = 14 m) ⇒ corrected at the data processing stage (s.):

$$\Delta t_R = -\frac{2}{c^2}\sqrt{a\mu}\;\; e \;\; \sin E$$

GPS control segment

- GPS control segment = 5 stations, master station at Colorado Springs
- Track satellites, computes and upload broadcast ephemerides into the satellites (broadcast ephemerides distributed to users via a “navigation message” included in the signal transmitted by the GPS satellites)
- Time synchronization on the satellites
- Monitors satellite “health”
- Decides and implements maneuvers when necessary

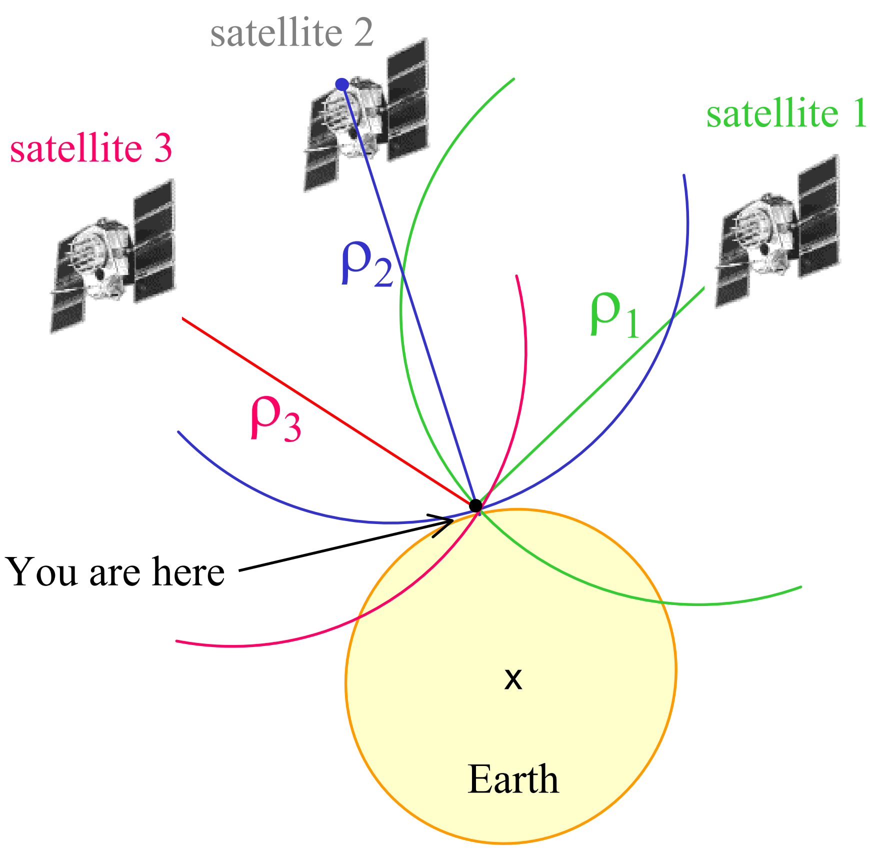


Global Positioning System (GPS) Master Control and Monitor Station Network

User segment

- GPS receivers
- All sizes, all prices
- For and endless variety of pplications…

GPS positioning: A simple principle



- Principle of GPS positioning:
  - Satellite 1 sends a signal at time t_{e1}
  - Ground receiver receives it signal at time t_r
  - The range measurement ρ_1 to satellite 1 is:
    - $\rho_1 = (t_r - t_{e1}) \times$ speed of light
    - We are therefore located on a sphere with radius ρ1 centered on satellite 1
  - 3 satellites $\Rightarrow$ intersection of 3 spheres

- In simple mathematical terms:

$$\rho_r^s = \sqrt{(X_s - X_r)^2 + (Y_s - Y_r)^2 + (Z_s - Z_r)^2}$$

  - GPS receivers:
    - Measure t_r
    - Decode t_e
    - Compute ρ_r^s
  - If the **position of the satellites in an Earth-fixed frame** *(X_s, Y_s, Z_s)* is known,
  - Then one can solve for *(X_r, Y_r, X_r)* (if at least 3 simultaneous range measurements)

- c = 299 792 458 m/s

Satellite-receiver time offset

- The receiver clocks are:
  1. Mediocre: stability ~10^{-5}-10^{-6} (~ crystal wrist watch)
  2. Not synchronized with the satellite clocks.
- There is a time difference between the satellite clocks (t_s) and the receiver clock (t_r): $\delta t = t_r - t_s$
  - The receivers therefore measures: $\tau = t + \delta t$
  - In terms of distance: $\tau \times c = (t + \delta t) \times c = r + \delta r = \rho$
  - The receiver actually measures ρ = **pseudorange**
- Practical consequences:
  - The time offset between satellite and receiver clocks is an additional unknown
  - We need 4 observations $\Rightarrow$ **4 satellites** visible at the same time
  - In order to compute a position, the receiver solves for δt => GPS receivers are **very precise clocks**! (Timing is a very important application of GPS)
  - δt is used by the receiver to synchronize its clock with the satellite clocks. That sync is as good as δt accuracy or ~ 0.1 μsec: we will still need to solve for δt

From the GPS signal to a position: Basic principle

- Measure arrival time of GPS signals from several satellites simultaneously
- Decode the GPS signal and figure out the signal propagation time (t_r-t_e), multiply by c = "pseudoranges" (= GPS data, or observables)
- Decode the navigation message and convert it into satellite positions
- Use at least 4 pseudoranges acquired at the same time from 4 different satellites to compute a position in an ECEF frame.
- Convert ECEF position into latitude-longitude-height in any geodetic system (for instance WGS84).

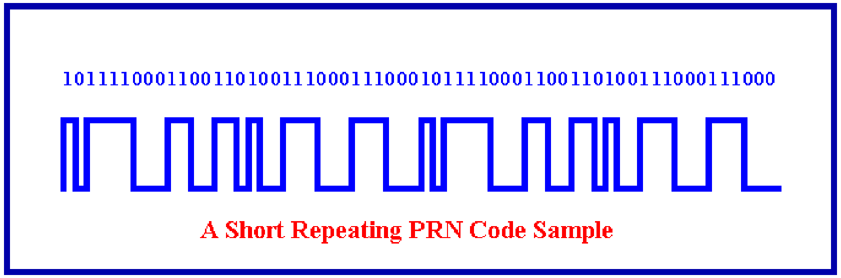
The GPS signal

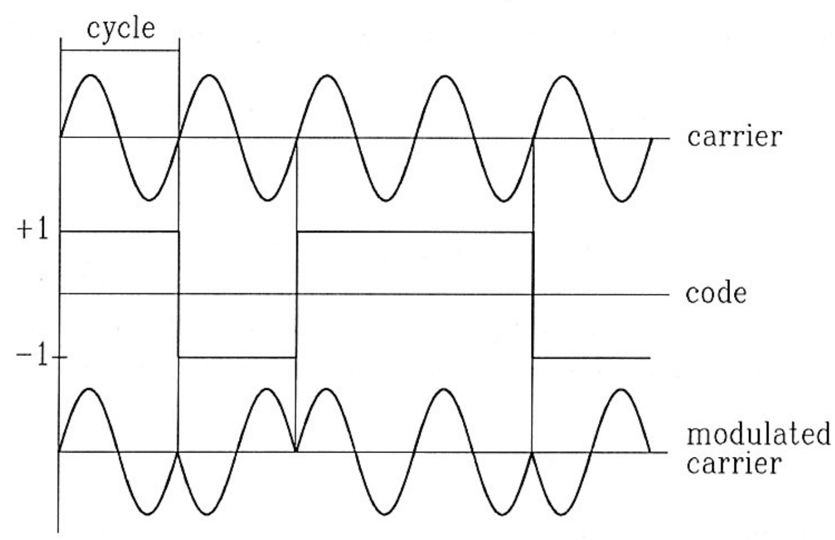
- The atomic clocks aboard the GPS satellites produce a fundamental frequency f_o = 10.23 Mhz

- Two frequencies are derived from it: **L1** *(fo x 154)* and **L2** *(fo x 120)*:
  - L1: 1.57542 GHz, wavelength 19.0 cm
  - L2: 1.22760 GHz, wavelength 24.4 cm
  - L1 and L2 are the **two carrier frequencies** used to transmit timing information by the GPS satellites
  - The information transmitted by the satellite is coded as a phase modulation of the carrier frequency

Phase modulation

- Information is coded as a sequence of +1/-1 (binary values 0/1), π shift in carrier phase when code state changes = biphase modulation
- Rate at which the phase shift occurs = chip rate
- **Pseudorandom noise" codes** (= PRN codes):
  - Unique to each satellite
  - Coarse Acquisition (C/A) code:
    - L1 only
    - Chip rate = 1023 MHz
  - Precision (P) code:
    - L1 and L2
    - Chip rate = 10.23 MHz
  - Encryption (W) code: encrypts the P-code into the Y-code (highly classified)





Biphase modulation of the GPS carrier phase

Navigation message

- **Navigation message**: ephemerides for all satellites, ionospheric correction parameters, system status, satellite clock offset and drift)
- Also coded by bi-phase modulation
- Chip rate = 50 bps
- 25 frames of 1500 bits each, divided into five 300 bits subframes
- 50 bps $\Rightarrow$ 300/50 = 6 sec to transmit one subframe, 6x5x25 = 750 sec (=12.5 min) to transmit an entire navigation message

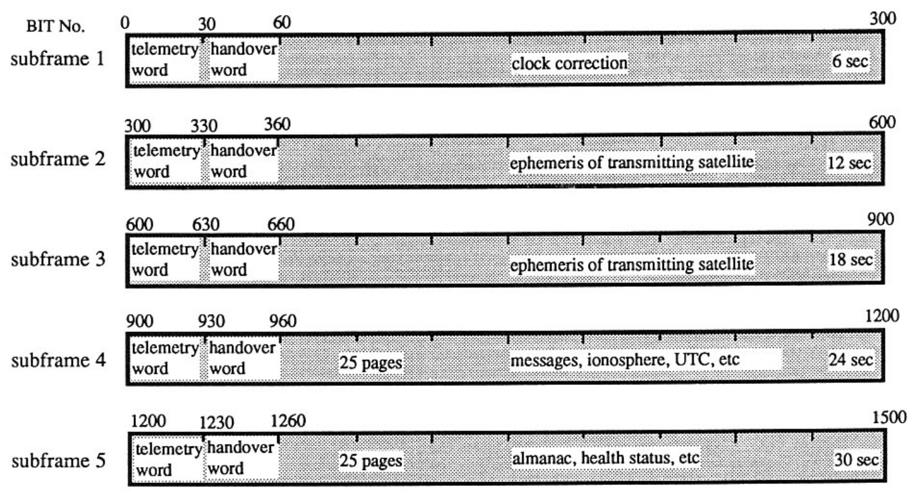


Figure 3.11 Structure of navigation message frame.

Receiver start-up

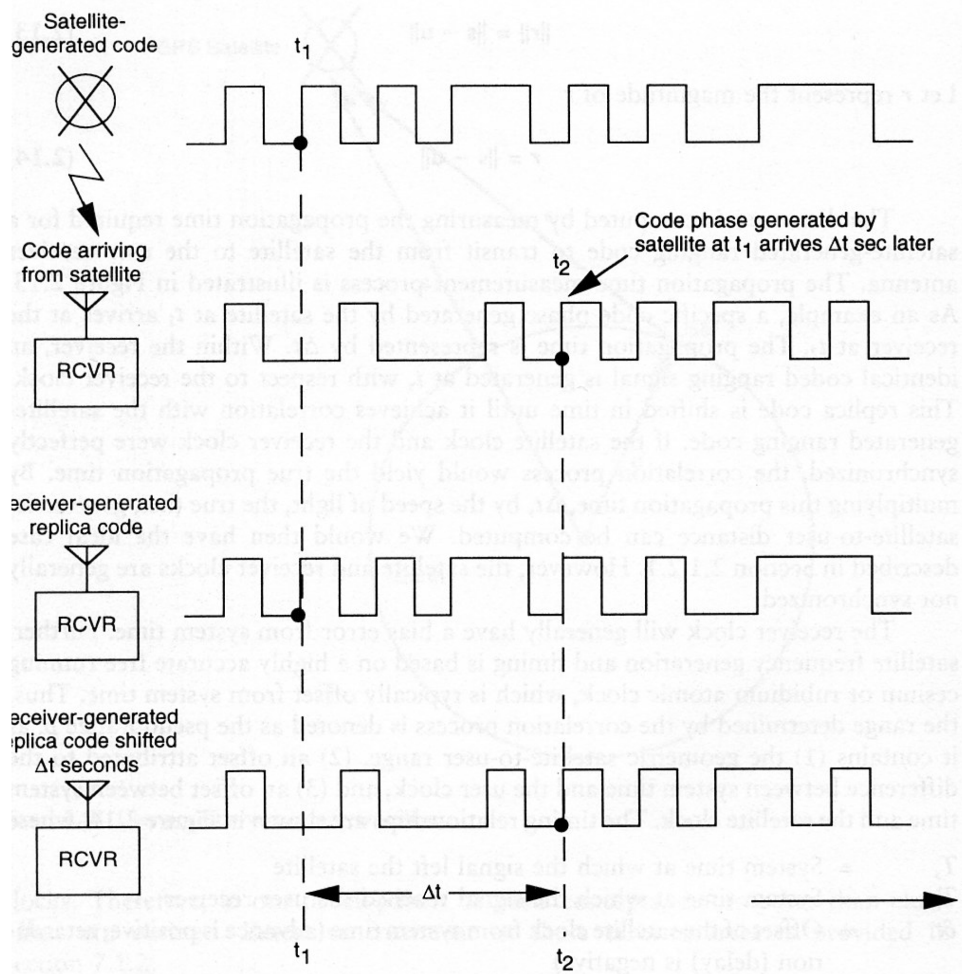
- General procedure:
    1. Acquire one satellite to get time and almanach
    2. Acquire 2 other satellites to get 2-D position
    3. Acquire 4^{th} satellite to get 3-D position
    4. Acquire any other visible satellite
- Time needed to get good position:
    - Hot start: few secs (rcv was off for a few secs: almanach ok, time ok, position close to last one)
    - Warm start: few mins (rcv was off for less than a day: clock ~ok)
    - Cold start: 10s of minutes (rvc was off for several days: time off, almanach expired, last position off)

Decoding in the receiver

- Radio frequency (RF) part of the receiver processes incoming signals:
  - L1 only (single-frequency receivers)
  - L1 and L2 (dual-frequency receivers)
- RF unit:
  - Processes incoming signal from different satellites in different channels (multichannels receivers, 4 to 12 channels)
  - Generates internal replica of the GPS signal:
    - Contains an oscillator (= clock) that generates L1 and L2 frequencies
    - Knows each PRN code (almost…)
  - Compares internally generated signal with incoming signal

Code measurements

- Code-correlation:
  - Shift of the internally generated signal in time until it matches the incoming one (receiver "locked" on a satellite)
  - **Time shift needed = signal travel time from satellite to receiver**



- Other techniques to retrieve phase information, independent of PRN codes:
  - Squaring: autocorrelation of the incoming signal
  - Cross-correlation: correlation between L1 and L2 using Y-code (Y-code is identical on L1 and L2)
  - Z-tracking: correlation on L1 and L2 using the P-code to obtain W-code
  - All these techniques have a lower SNR than the code-correlation:
    - Squaring: -30 dB
    - Cross correlation: -27 dB
    - Z-tracking: -14 dB

Code measurements

- GPS receivers measure pseudoranges ${}^{j}R_i(t)$, that can be modeled as:

$$ {}^{j}R_i(t) = {}^{j}\rho_i(t) + c({}^{j}\delta(t) - \delta_i(t)) + \Delta I(t) + \Delta T(t) + MP(t) + \varepsilon $$

t = time of epoch
${}^{j}R_i$ = pseudorange measurement
${}^{j}\rho_i$ = satellite-receiver geometric distance
c = speed of light
${}^{j}\delta$ = satellite clock bias
δ_i = receiver clock bias
ΔI = ionospheric propagation error
ΔT = tropospheric propagation error
MP = multipath
ε = receiver noise
(ranges in meters, time in seconds)

- ΔI and ΔT are correction terms because GPS signal propagation is not in a vacuum (more later)

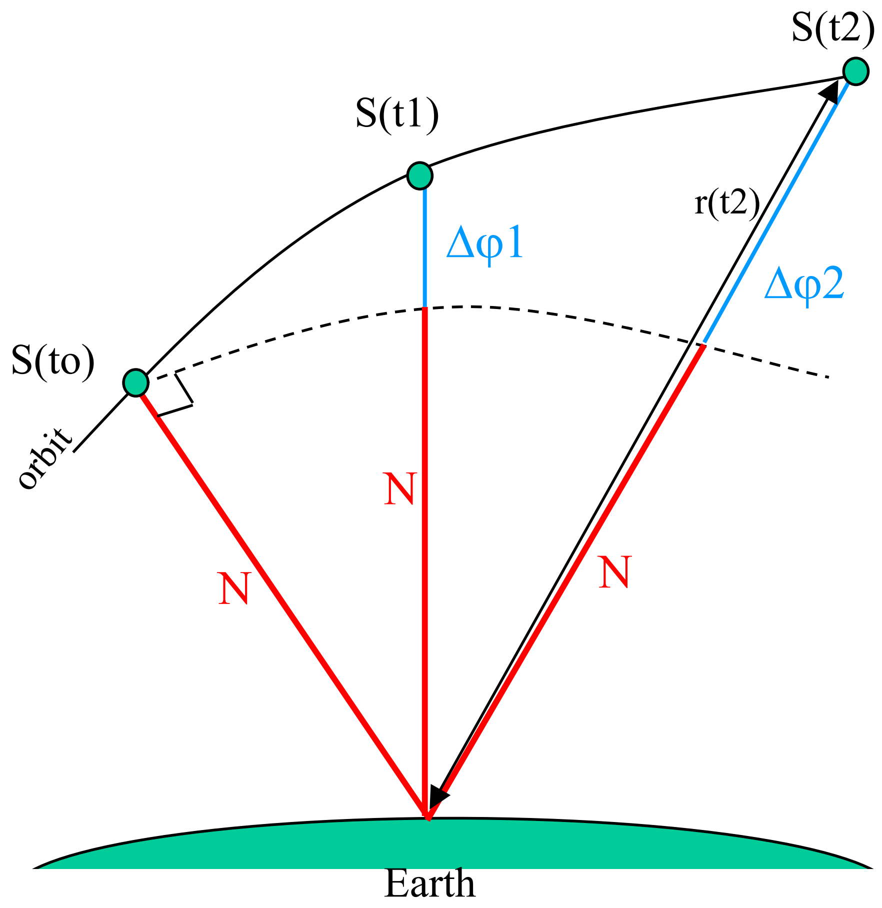
- MP = multipath noise, reflection of GPS signal off surfaces near antenna (more later)

Pseudorange noise

- Correlation function width: The width of the correlation is inversely proportional to the bandwidth of the signal.
  - C/A code = 1 MHz bandwidth $\Rightarrow$ correlation produces a peak 1 msec wide = 300 m
  - P code = 10 MHz bandwidth $\Rightarrow$ correlation produces 0.1 msec peak = 30 m
- Rough rule: Peak of correlation function can be determined to 1% of width (with care).
  - Range accuracy = 3 m for C/A code
  - Range accuracy = 0.3 m for P code
- Pseudorange measurements = low accuracy but absolute

Phase measurements

- When a satellite is locked (at t_o), the GPS receiver starts tracking the incoming phase
- It counts the (real) number of phases as a function of time = $\Delta\varphi\,(t)$
- But the initial number of phases N at t_o is unknown…
- However, if no loss of lock, N is constant over an orbit arc



Phase measurements

- Geometrical interpretation:

$$\Delta\Phi = \frac{R}{\lambda} - N$$

$$R = \rho + c\delta t$$

$$\Rightarrow \Delta\Phi = \frac{\rho}{\lambda} + \frac{c}{\lambda}\delta t - N$$

$\Delta\Phi$ = phase measurement
R = pseudorange
c = speed of light
ρ = geometric range
λ = wavelength
δt = sat-rcv clock offset
N = phase ambiguity

- The phase equation (units of cycles):

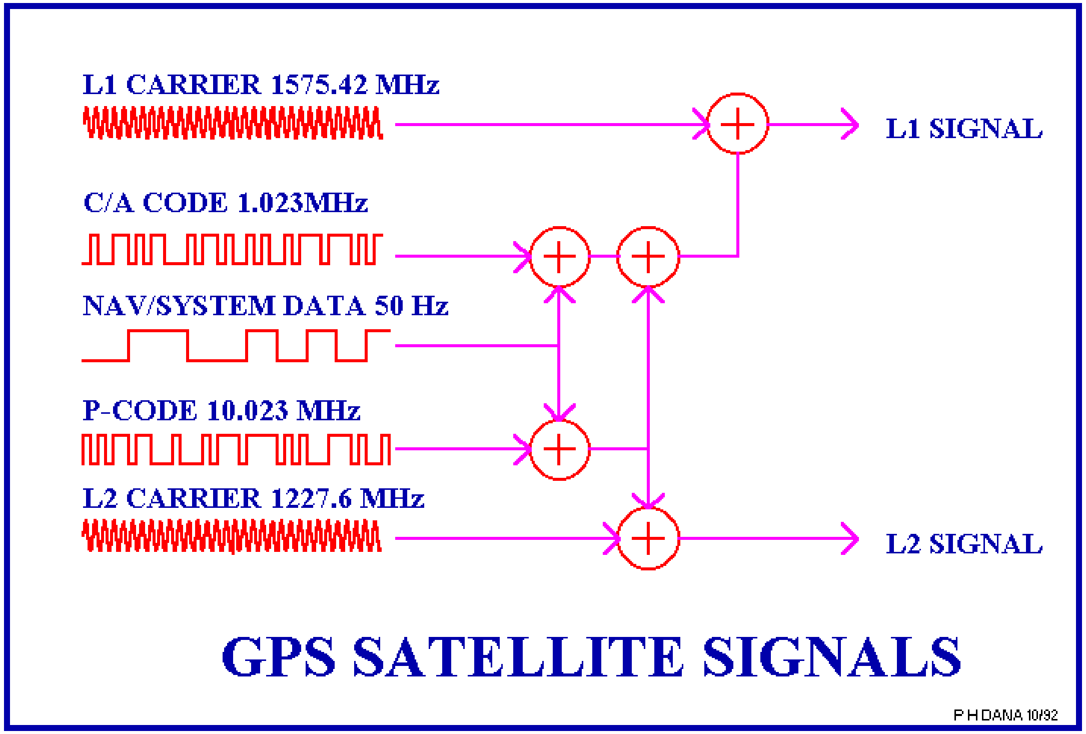
$$\Phi_i^k(t) = \rho_i^k(t) \times \frac{f}{c} + \left(h^k(t) - h_i(t)\right) \times f + ion_i^k(t) + trop_i^k(t) - N_i^k + \varepsilon$$

t = time of epoch
i = receiver, k = satellite
ρ_i^k = geometric range
h^k = satellite clock error, h_i = receiver clock error
ion_i^k = ionospheric delay, $trop_i^k$ = troposp
N_i^k = phase ambiguity, ε = phase noise

Phase measurements

- Phase can be converted to distance by multiplying by the wavelength $\Rightarrow$ phase measurements are another way for measuring the satellite-receiver distance
- Phase can be measured to ~1% of the wavelength $\Rightarrow$ range accuracy 2 mm for L1, 2.4 mm for L2
- Phase measurements are very precise, but ambiguous
- To fully exploit phase measurements, one **must** correct for propagation effects (several meters)

GPS observables



- GPS receivers can record up to 5 observables :
  - φ1 and φ2: phase measurements on L1 and L2 frequencies, in cycles
  - C/A, P1, P2: pseudorange measurements, in meters
- Plus Doppler phase = dφ/dt

GPS observables

- GPS observables stored in receivers in binary proprietary format
- Receiver Independent Exchange format (RINEX) = ASCII exchange format
- Format description: ftp://igscb.jpl.nasa.gov/igscb/data/format/rinex2.txt
- Conversion from binary proprietary to RINEX:
  - Proprietary software
  - Freewares: *e.g.* teqc (www.unavco.ucar.edu)

RINEX observation file

```
 2.00           OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
teqc  1999Jul19     CNRS_UMRGA          20021201 12:04:20UTCPGM / RUN BY / DATE
Solaris 2.3|S-Sparc|cc SC3.0|=+|*Sparc                      COMMENT
BIT 2 OF LLI FLAGS DATA COLLECTED UNDER A/S CONDITION       COMMENT
SJDV                                                        MARKER NAME
10090M001                                                   MARKER NUMBER
___                 REGAL                                   OBSERVER / AGENCY
845                 ASHTECH Z-XII3      CD00                REC # / TYPE / VERS
317                 ASH700936A_M    NONE                    ANT # / TYPE
  4433469.9683   362672.6919  4556211.6229                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     5    L1    L2    C1    P1    P2                        # / TYPES OF OBSERV
    30.0000                                                 INTERVAL
Forced Modulo Decimation to 30 seconds                      COMMENT
 SNR is mapped to RINEX snr flag value [1-9]                COMMENT
  L1: 1 -> 1;  90 -> 5; 210 -> 9                            COMMENT
  L2: 1 -> 1; 150 -> 5; 250 -> 9                            COMMENT
  2002    11    30     0     0   30.0000000     GPS         TIME OF FIRST OBS
                                                            END OF HEADER
 02 11 30  0  0 30.0000000  0  8G14G 7G31G20G28G 1G25G11
  -7096034.24049  -5509904.97345  23971309.103    23971309.038    23971310.842
 -12570276.74149  -9768618.40046  23379169.469    23379168.448    23379172.496
  -4157689.84249  -3201324.38045  24195891.298    24195890.733    24195894.168
 -25480193.34249 -19826614.77248  20670858.774    20670857.983    20670861.191
  -5589280.20049  -4319738.39345  24553697.713    24553697.259    24553700.349
 -10252537.24449  -7918950.15946  23060092.127    23060091.841    23060095.687
  -4143445.15949  -2509987.53445  24581180.488    24581179.713    24581183.992
 -29659606.34049 -23089397.33548  20312382.965    20312382.530    20312384.719
 02 11 30  0  1  0.0000000  0  8G14G 7G31G20G28G 1G25G11
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  -4043324.79449  -3112208.77545  24217654.165    24217653.747    24217658.209
 -25518762.53849 -19856668.69248  20663519.280    20663518.524    20663521.550
  -5521754.77149  -4267121.22845  24566547.413    24566547.593    24566550.660
 -10357839.61649  -8001003.94446  23040053.767    23040053.443    23040058.358
  -4207531.87749  -2559925.21345  24568984.944    24568985.325    24568989.371
 -29640011.07349 -23074128.30548  20316111.836    20316111.559    20316113.648
```

Header

Data blocks:

Range in meters

Phase in cycles

GPS observables: Summary

- **Pseudorange measurements (C/A, P1, P2):**
  - Geometric range + clock offset + noise:

$$\rho = r + \Delta t \times c$$

  - Accuracy of pseudorange measurements by GPS receivers ~ 1% of correlation peak width:
    - 3 m with C/A code
    - 0.3 m with P code
  - Low accuracy but absolute measurements
- **Phase measurements (L1, L2):**
  - Geometric range + clock offset - initial phase ambiguity N:

$$\varphi = r \times f/c + \Delta t \times f - N$$

  - Accuracy of phase measurements in GPS receivers ~ 0.005 cycle (0.005 x 20 cm = 0.2 mm) $\Rightarrow$ millimeter accuracy theoretically possible
  - Very accurate measurements but ambiguous