Code No: 44102 **Set No. 1**

II B.Tech II Semester Regular Examinations, Apr/May 2009
MATHEMATICS FOR AEROSPACE ENGINEERS
(Aeronautical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Prove that $\beta(m,n) = \int_0^1 \frac{\left(x^{m-1}+x^{n-1}\right)}{(1+x)^{m+n}}dx$
   (b) Show that $\int_0^\infty x^m\, e^{-ax^n}dx = \frac{1}{n\,a^{\frac{(m+1)}{n}}}\,\Gamma\left(\frac{m+1}{n}\right)$ Where m and n are Positive constants
   (c) Show that $\int_0^\infty x^n e^{-a^2x^2}dx = \frac{1}{2a^{n+1}}\left|\left(\frac{n+1}{2}\right)\right.$ $n > -1$ and deduce that $\int_0^\infty \cos x^2 dx = \int_0^\infty \sin x^2 dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$ [5+5+6]

2. (a) If w = f(z) is an analytic function of z such that $f^1(z) \neq 0$. prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log f(z) = 0$
   (b) Prove that $f(z) = \sqrt{xy}$ is not analytic at the origin even though the C-R equations are satisfied at the origin. [8+8]

3. (a) Evaluate the equation $\int_c \frac{\left(z^2-z-1\right)}{z(z-1)^2}dz$ with c : $\left|z-\frac{1}{2}\right| = 1$ using Cauchy's integral formula.
   (b) Using Cauchy's integral formula, evaluate $\int_c \frac{e^{2z}}{(z^2+\pi^2)^3}dz$ where c is $|z| = 4$
   (c) Evaluate $\int_{(0,0)}^{(1,1)}(3x^2+4XY+ix^2)dz$ along $y = x^2$ [5+6+5]

4. (a) Show that
      i. $\frac{1}{z^2} = 1 + \sum_{n=1}^{\infty}(n+1)(z+1)^n$ when $|z+1| < 1$
      ii. $\frac{1}{z^2} = \frac{1}{4} + \frac{1}{4}\sum_{n=1}^{\infty}(-1)^n(n+1)\left(\frac{z-2}{2}\right)^n$ when $|z-2| < 2$
   (b) Expand f (x) = $\frac{-1}{(z-1)(z-2)}$ as a power series in z in the regions
      i. $1 < |z| < 2$
      ii. $|z| > 2$ [8+8]

5. (a) Find the image of the semi -infinite strip x.>0, 0<y<2 under the transformation w = iz+1
   (b) Establish the bilinear transformation which maps the points z = 1, i, -1 into the points w = i, 0,-i respectively. [8+8]

6. (a) What is summation convention in tensor analysis? Explain Write the following in using summation convention
      i. $(x^1)^1+(x^1)^2+(x^1)^3+ \ldots (x^1)^n$
      ii. $(x^1)^2+(x^2)^2+(x^3)^2+ \ldots (x^n)^2$

(b) Define Christoffel symbol of first and second kind. If $(ds)^2 = (dr)^2 + r^2\ (d\theta)^2 + r^2 \sin^2\theta\ (d\varphi)^2$, then find the value of [13,3]and $\left[{}^{3}_{13}\right]$ [8+8]

7. (a) Define the axioms of probability. Two six faced unbiased dice are thrown. Find the probability that the sum of numbers shown is either 7 or their product is 12.

(b) A firm producing glass bottles has three factories producing 25%, 35% and 40% of it's total output .The corresponding percentage of defectives in three factories are 5,4, and 2 respectively. A customer brings in a bottle purchased from the firm which was found to be defective. Find the probabilities that it was produced at each of the three factories. [8+8]

8. (a) The mean yield for one acre plot is 662 kgs with standard deviation 32 .Assuming normal distribution, how many one acre plots in a batch of 1000 plots are expected to yield

   i. over 700 kgs.
   ii. below 650 kgs.
   iii. what is the lowest yield of the best 100 plots.

(b) Show that if X (t) and Y(t) are two random processes and $R_{XY}(\tau\ )$ and $R_{YX}\ (\tau)$ are their respective auto correlation functions, then, $R_{XY}(\tau)| \leq \sqrt{[R_{XX}(0) + R_{YY}(0)]}$ holds. [8+8]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) Prove that $\int_{-1}^{1}(1-x^2)\,P_m'(x)\,P_n'(x)\,dx = \frac{2n(n+1)}{2n+1}$ Where m and n are Positive integers and m=n
   (b) Prove that $(2n+1)\,(x^2-1)\,P'{}_n(x) = n(n+1)\,[P_{n+1}(x) - P_{n-1}(x)]$
   (c) Prove that $J''0\,(x) = \frac{1}{2}\,[J_2\,(x) - J_0(x)]$ [6+5+5]

2. (a) Show that f(z) = $\begin{cases} \frac{\mathrm{xy}^2(x+iy)}{x^2+y^2}, & z \neq 0 \\ = 0\,if\ z = 0, & is\,not\,analytic, \end{cases}$ at z=0 although C-R equations are satisfied at the origin
   (b) Separate the real and imaginary parts of sin z. [8+8]

3. (a) Evaluate $\int_0^{1+i}(x^2 - iy)\,dz$ along the paths.
      i. y=x
      ii. y=x^2.
   (b) Evaluate $\int\limits_c \frac{(e^z\ \sin 2z-1)dz}{z^2(z+2)^2}$ where c is $|z| = 1/2$ using Cauchy's integral formula. [8+8]

4. (a) Find the poles and the corresponding residues of $\frac{z^3}{(z-1)(z-2(z-3))}$
   (b) Evaluate $\int_c \frac{(3z-4)}{z(z-1)(z-2)}dz$ by residue theorem where c is $|z| = 3$ [8+8]

5. (a) Find the image of
      i. the infinite strip 1 < x < 2
      ii. |z + 1| = 1 under the transformation w =1 /z.
   (b) Find the bilinear transformation which maps the points z = 1, i, - 1 into the points w = i. 0, -i respectively. Hence find the image of $|z| < 1$ [8+8]

6. (a) i. Explain summation notation in tensor analysis
      ii. Define Knonecker delta.Evaluate $\delta_j^i\ \delta_k^j$
   (b) Define Christoffel symbol of first kind. If (ds)^2 = (dr)^2 +r^2 (dθ)^2 + r^{2}sin^2 θ (dφ)^2, then find the value of [22,1]and [13,3] [8+8]

7. (a) A person takes 4 tests in succession. The probability of his passing the first test is p, while that of his passing each succeeding test is p or p/2 according as he passes or fails in the preceding test. He qualifies provided he passes at least three tests. Find the probability of his qualifying.

(b) A consulting firm rents cars from three agencies in the following manner. 20% of cars from agency D, 20% of cars from agency E, 60% of cars from agency F. If 10% of the cars from D, 12% of the cars from E and 4% of the cars from F have bad tyres. If a car received by the is found have bad tyres, what is the probability that the car was supplied by the agency F? [8+8]

8. (a) Given the probability density function

x:	0	1	2	3	4	5	6
p(x):	k	3k	5k	7k	9k	11k	13k

find k, P(X<4) P(X>3), P(3 <X $\leq$ 4)

(b) The average life of a bulb is 1200 hours and standard deviation is 200 hours. If X is the life period of a bulb which is distributed normally, find the probability that a randomly picked bulb will last

i. less than 600 hours
ii. more than 800 hours
iii. between 1100 and 1300 hours. [8+8]

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Code No: 44102 **Set No. 3**

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MATHEMATICS FOR AEROSPACE ENGINEERS
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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) Show that $\int_0^1 \frac{x^n}{\sqrt{1-x^2}}dx = \frac{2.4.6----------(n-1)}{1.3.5.----------n}$ Where n is an odd integer.
   (b) Evaluate $\int_0^1 \frac{dx}{\sqrt{-\log x}}$ using Γ function
   (c) Prove that $[(x)\ [(1-x) = \frac{\pi}{\sin x\pi}$ [5+5+6]

2. (a) If w=f(z)=u+iv is an analytic function and ϕ is any function of x and y with partial derivatives of first and second orders, than prove that $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \left\{\frac{\partial^2\phi}{\partial u^2} + \frac{\partial^2\phi}{\partial v^2}\right\}\left|f'(z)\right|^2$
   (b) If f(z)=u+iv is an analytic function of z and if u-v=$(x-y)\left(x^2+4xy+y^2\right)$, find f(z) in terms of z. [8+8]

3. (a) Evaluate $\int_c \frac{\left(4z^2-4z+1\right)}{(z-2)^2(z^2+4)}dz$ where C is $|z| = 3$ by using Cauchys integral formula.
   (b) Evaluate $\int_{(0,1)}^{(2,5)}(3x+y)$ dx + (2y-x) dy along
      i. The curve $y = x^2 + 1$
      ii. The straight line joining (0,1) and (2,5). [8+8]

4. (a) Determine the poles of the function f(z) and residue at each pole where $f(z) = \frac{\left(z^2-2z\right)}{(z+1)^2(z^2+1)}$
   (b) Evaluate $\int_c \frac{(12z-7)}{(z-1)^2(2z+3)}dz$ by Cauchy's residue theorem where C is the circle $|z| = 2$ [8+8]

5. (a) What is the region of the w- plane in to which the region in z- plane bounded by the lines x =0 ,y =0 .x =1 , y = 2 under the transformation w =z +2 - i
   (b) Find the bilinear transformation which maps the points z = 2 , -2 ,2 into the points w = 0, i, - i respectively. What are the critical points of such transformation? [8+8]

6. (a) Explain summation notation in tensor analysis with suitable examples. Write the terms contained S =a_{ij} x^i x^j taking n =3
   (b) Define Christoffel symbol of first kind. If $(ds)^2 = r^2\ (d\theta)^2 + r^2\sin^2\theta\ (\ d\varphi)\ 2$, then find the value of [22, 1] [8+8]

7. (a) 500 people were asked about their morning vitamin intake. It was found that 150 Take vitamin B, 200 take vitamin C, 165 take vitamin E, 57 take both B and C, 125 take both B and E, 82 take all three vitamins. What is the probability that a person takes none of the vitamins?

(b) A bag contains 5 red, 3 blue and 4 black balls. If three balls are drawn at random, what is the probability that

i. the three balls are of different colours

ii. two balls are of the same colour

iii. all balls are of same colour. [8+8]

8. (a) For the distribution dF = K e $^{-x/\sigma}$ dx , 0< x < ∞ , σ> 0 where K is constant, show that mean = standard deviation = σ

(b) Find the mean and variance of Poisson distribution. [8+8]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) Show that $\int_{-1}^{1} x^2 P_{n+1}(x) P_{n-1}(x)\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$
   (b) prove that $\frac{d}{dx}(xJ_n J_{n+1}) = x\left(J_n^2 - J_{n+1}^2\right)$. [8+8]

2. (a) Prove that f $(x, y) = \frac{x^2 y(y-x)}{(x^6+y^2)(x+y)}, (x, y) \neq (0, 0) = 0$ if (x,y))=(0,0) Is discontinuous at (0,0).
   (b) Find f(z)=u+iv given that
   $u + v = \frac{\sin 2x}{\cosh 2y - 2 \cos 2x}$ [8+8]

3. (a) Evaluate $\int_{1-i}^{2+3i} (z^2 + z)dz$ along the straight line joining the points (1,-1) and (2,3)
   (b) Using Cauchy's integral formula evaluate $\int\limits_c \frac{(z+4)dz}{z^2+2z+5}$ where c is $|z + 1 - i| = 2$. [8+8]

4. (a) Expand $\frac{1}{(z^2+1)(z^2+2)(z^2+3)}$ in Positive and negative power of z if $1 < |z| < \sqrt{2}$
   (b) Expand f(z) = sin z in Taylor's series about $z = \frac{\pi}{4}$ and find the region of convergence. [8+8]

5. (a) Find the image of the strip - $\pi/2 <$ x $<\pi/2$, $1 <$ y < 2 under the mapping w = sin z
   (b) Find the bilinear transformation which maps the points 1, - i, 2 of the z-plane into 0, 2 ,-i of the w-plane. Find the invariant points of the transformation. [8+8]

6. (a) If the components of two tensors are equal in one coordinate system, show that they are equal in all coordinate systems.
   (b) Define Christoffel symbol of first kind. Prove that $\partial g_{ij}/\partial x^k = [ik, j] + [jk, i]$ [8+8]

7. (a) Urn I contains 2 white and one red and 3 black balls. Urn II contains one white and 3 red and 2 black balls. An urn is selected and a ball is drawn at random from it. Give the sample space.
   (b) Show that, if A and B are independent events then A^c and B^c are also independent .

8. (a) Given the probability density function

x:	0	1	2	3	4	5	6
p(x):	k	3k	5k	7k	9k	11k	13k

find k, P(X<4) P(X>3), P(3 <X $\leq$ 4)

(b) The average life of a bulb is 1200 hours and standard deviation is 200 hours. If X is the life period of a bulb which is distributed normally, find the probability that a randomly picked bulb will last

i. less than 600 hours
ii. more than 800 hours
iii. between 1100 and 1300 hours. [8+8]

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